

## AGRICULTURE POTFOLIO MODEL MODEL TWO

Keywords: Decision making under uncertainty, efficient portfolio, variance analysis, MOTAD

### DATA

Net income from three crops per acre of land  
(Income in thousand dollar per acre of land) Excel output

<b>Year</b>	<b>X1</b>	<b>X2</b>	<b>X3</b>
2003	3.40	11.20	12.50
2004	6.20	5.32	9.50
2005	7.50	12.45	10.30
2006	4.50	15.23	12.30
<b>Mean income</b>	<b>5.4</b>	<b>11.05</b>	<b>11.15</b>
<b>Standard deviation of net income</b>	<b>1.81</b>	<b>4.17</b>	<b>1.48</b>

SPSS output

#### Descriptive Statistics

	Mean	Std. Deviation	N
X1	5.4000	1.81292	4
X2	11.0500	4.17484	4
x3	11.1500	1.48212	4

SPSS output

**Correlations**

		X1	X2	x3
X1	Pearson Correlation	1	-.251	<b>-.849</b>
	Sig. (2-tailed)		.749	.151
	Sum of Squares and Cross-products	9.860	-5.706	-6.840
	Covariance	3.287	-1.902	<b>-2.280</b>
	N	4	4	4
X2	Pearson Correlation	-.251	1	.715
	Sig. (2-tailed)	<b>.749</b>		.285
	Sum of Squares and Cross-products	-5.706	52.288	13.274
	Covariance	-1.902	17.429	4.425
	N	4	4	4
x3	Pearson Correlation	<b>-.849</b>	.715	1
	Sig. (2-tailed)	.151	.285	
	Sum of Squares and Cross-products	-6.840	13.274	6.590
	Covariance	<b>-2.280</b>	4.425	2.197
	N	4	4	4

Input requirement to produce per acre of land by three crops

	<b>X1</b>	<b>X2</b>	<b>X3</b>	<b>Availability</b>
Labour hour	12	10	5	5000 labor hour
Fertilizer	10	5	8	2000 kg
Pesticide	2	4	10	3000 kg
Total Land				2000 acre

- The target is to minimize the risk as well as maximize the income from the whole business under risk and uncertainty.
- Target is to build a minimum risk portfolio using MOTAD.

- MOTAD is an alternative to Quadratic Programming to build minimum risk efficient frontier.
- Lingo application.
- Lingo and Lingo will provide the same result.
- Xi is acre of land

### **SOME COMMANDS OF LINDO**

Edit  
Cat  
Retr  
Solution  
Go  
Alter  
Del  
Edit  
Stat  
go

## **LINEAR PROGRAMMING SOLUTION USING LINDO MAXIMIZATION OF INCOME**

Maximization of income from three crops

$$\text{Max } 5.40 X_1 + 11.05 X_2 + 11.15 X_3$$

Subject to

Land.

$$X_1 + X_2 + X_3 \leq 2000$$

Labor hour

$$12X_1 + 10X_2 + 5X_3 \leq 5000$$

Fertilizer

$$10X_1 + 5X_2 + 8X_3 \leq 2000$$

Pesticide

$$2X_1 + 4X_2 + 10X_3 \leq 3000$$

Non-negativity constraint

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

### **SOLUTION**

Maximum income can be achieved: 4420 dollar (We can not get income more than this from this particular agri-business)

$X_1=0$  acre,  $X_2= 400$  acre,  $X_3=0$  acre should be planted to achieve highest income.

Idle land = 1600 acre

## MINIMIZATION OF RISK OR STANDARD DEVIATION OF PORTFOLIO

We need MOTAD (Minimization of total absolute deviation) technique to minimize the risk of a portfolio.

### MOTAD SETTING WHEN THREE CROPS IN A PORTFOLIO USING LINDO

Minimization of the total absolute deviations (MOTAD)

$$\text{MIN } P03 + P04 + P05 + P06 + N03 + N04 + N05 + N06$$

Subject to

Expected income from the business

$$5.40 X_1 + 11.05 X_2 + 11.15 X_3 = 4420 \text{ (maximum income)}$$

Subject to

Land.

$$X_1 + X_2 + X_3 \leq 2000$$

Labor hour

$$12X_1 + 10X_2 + 5X_3 \leq 5000$$

Fertilizer

$$10X_1 + 5X_2 + 8X_3 \leq 2000$$

Pesticide

$$2X_1 + 4X_2 + 10X_3 \leq 3000$$

Risk Rows : 2003 to 2006

$$-P03 + N03 - 2.0x_1 + 0.15x_2 + 1.35x_3 = 0$$

$$-P04 + N04 + 0.8x_1 - 5.73x_2 - 1.65x_3 = 0$$

$$-P05 + N05 + 2.1x_1 + 1.4x_2 - 0.85x_3 = 0$$

$$-P06 + N06 - 0.9x_1 + 4.18x_2 + 1.15x_3 = 0$$

Non- negativity constraints

$$X_1 \geq 0$$

$$X_2 \geq 0$$

$$X_3 \geq 0$$

## RISK OF THREE CROP PORTFOLIO

$$\text{Variance of a portfolio} = \sum_{j=1}^n \sum_{k=1}^n X_j X_k \sigma_{jk}$$

Variance of three crops portfolio.

$$\begin{aligned} \text{Variance} = & X_1 X_1 \sigma_{11} + X_1 X_2 \sigma_{12} + X_1 X_3 \sigma_{13} \\ & X_2 X_1 \sigma_{21} + X_2 X_2 \sigma_{22} + X_2 X_3 \sigma_{23} \\ & X_3 X_1 \sigma_{31} + X_3 X_2 \sigma_{32} + X_3 X_3 \sigma_{33} \end{aligned}$$

Here  $n=3$  (variances as well as number of assets)

$$= n + (n^2 - n)$$

$$= 3 + (9 - 3) = 9$$

Here, 3 variances and 6 co-variances of income.

Where,

$V$  is the variance of income of the portfolio

$X_j$  is the amount of land allocated or decision variable.

$\sigma_{jk}$  is the covariance of income between the  $j$ th and  $k$ th enterprises.

$\rho_{jk}$  is the correlation coefficient between  $j$ th and  $k$ th enterprise.

$\sigma_{jj}$  is the variance of income of the  $j$ th enterprise or covariance of the  $j$ th variable with itself.

$\sigma_j$  is the standard deviation of income of the  $j$ th enterprise.

$\sigma_k$  is the standard deviation of income of the  $k$ th enterprise.

Variance of three crops portfolio

$$= X_1 X_1 \sigma_{11} + 2 X_1 X_2 \sigma_{12} + 2 X_1 X_3 \sigma_{13} + X_2 X_2 \sigma_{22} + 2 X_2 X_3 \sigma_{23} + X_3 X_3 \sigma_{33}$$

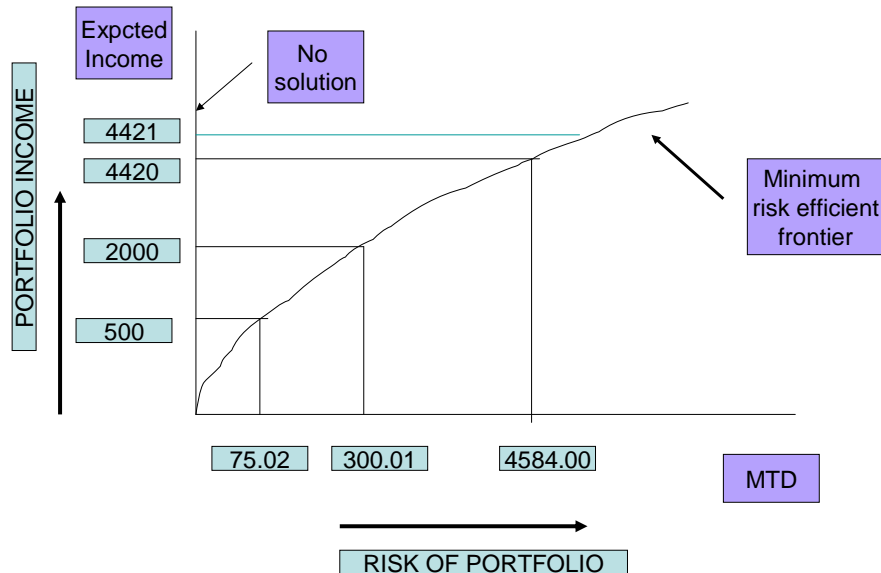
## TASK

Calculate the variance and standard deviation of three crops and put the result in the Agri-Business Plan below.

## AGRI-BUSINESS PLAN

<b>Efficient Plans</b>	<b>I</b>	<b>II</b>	<b>III</b>	<b>IV</b>
Expected income from the whole business	500	2000	4420 (Highest risk and maximum income plan)	4421 (No feasible solution as the expected income exceeded maximum income)
Standard deviation of three crop portfolio	<b>28.54</b>	<b>113.6</b>	<b>1669.9</b>	
Minimum Total Deviation (MTD)	75.02	300.01	4584.00	
X1 (acre of land)	22.81	91.24	0 (LP solution) 0 (Motad solution)	
X2 (acre of land)	0.0	0.00	400 (LP Solution) 400 (Motad solution)	
X3 (acre of land)	33.79	135.18	0 (LP solution) 0 (Motad solution)	
Total idle land (acre of land)	1943.39	1773.6	1600	

## Expected Income and MTD Frontier



- Since covariance or correlation between X1 and X3 is negative and high, so they are appearing in plan I and Plan II. As the income and the standard deviation of income for X2 is high, so it will not appear in Plan I or II but in plan III, the highest risk and income plan.
- Plan I and Plan II are low expected income plan so less risky crops are likely to appear in these two plans.
- Motad model will also consider input requirements by three crops. Less input requirement will be preferred in the plan.

## GUIDELINE

- The target of MOTAD model is to reduce the variance of the portfolio.
- MOTAD model will be looking for those crops in the efficient plan which are negatively correlated or less correlated or negatively covariate or less covariate to reduce the variance of the three crops portfolio.



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