QUADRATIC AND MOTAD PROGRAMMING
MODEL THREE

Key words: Linear Programming, MOTAD, Quadratic Programing, Variance and Covariance Matrix, Coefficient of Variation, Standard Deviation, Correlation analysis, Portfoilio Analysis, Markowitz Portfolio, LINGO, LINDO.

Estimation of variance and co-variance matrix

Before a complete model can be constructed, the estimation on variance and covariances are needed. There are a number of ways for measuring the variance. In this study following formula has been employed for measuring the variance of return of both prevailing and efficient farm plans given in equation (1.1).

$$V = \sum_{j=1}^{n} \sum_{k=1}^{n} X_j X_k \sigma_{jk} \qquad (1.1)$$

The variance of the five crops portfolio is stated as follows :

$$V = X_1X_1\sigma_{11} + 2X_1X_2\sigma_{12} + 2X_1X_3\sigma_{13} + 2X_1X_4\sigma_{14} + 2X_1X_5\sigma_{15} + X_2X_2\sigma_{22} + 2X_2X_3\sigma_{23} + 2X_2X_4\sigma_{24} + 2X_2X_5\sigma_{25}$$

$$+ X_3X_3\sigma_{33} + 2X_3X_4\sigma_{34} + 2X_3X_5\sigma_{35} + X_4X_4\sigma_{44} + 2X_4X_5\sigma_{45} + X_5X_5\sigma_{55}$$

$$= \sum_{j=1}^{5} X^2_j \sigma_{jj} + \sum_{j=1}^{5} \sum_{k=1}^{5} X_j X_k \sigma_{jk}$$

$$= \sum_{j=1}^{5} X_j^2 \sigma_{jj} + \sum_{\substack{j=1 \\ \text{for } j \neq k}}^{5} \sum_{k=1}^{5} X_j X_k \rho_{jk} \sigma_j \sigma_k \qquad \text{since} \quad \sigma_{jk} = \rho_{jk} \sigma_j \sigma_k$$

Where,

V is the variance of gross returns of the enterprise portfolio

X_j is the the prevailing farm acreage as well as efficient farm acreage.

σ_{jk} is the covariance of gross returns between the jth and kth enterprises.

ρ_{jk} is the correlation coefficient between jth and kth enterprise.

σ_{jj} is the variance of the jth enterprise or covariance of the jth variable with itself.

σ_j is the standard deviation of the jth enterprise.

The variation in prices and correlation coefficient of returns of the above are mainly determined by the market forces and are independent of the producer‘s production condition. But the allocation of land area in favor of less risky enterprises may reduce the portfolio risk considerably. However, the variance and co-variance of matrix of gross return for crop activities are exhibited in Table 1.2 for Dhaka division.

Table 1.1: Measurement of Expected Gross Return by Crop Activities in Dhaka Division.

(Gross Return in Million Taka Per '000 Acres)

Year	Cereals	Pulses	Oilseeds	Fibre	Vegetables
1983 - 84	3.480	0.625	1.218	3.897	7.055
1984 - 85	4.053	0.651	1.15	7.082	6.847
1985 - 86	3.967	0.984	1.379	4.363	8.179
1987 - 88	4.585	1.958	2.767	6.413	6.635
1988 - 89	4.797	3.476	2.528	4.796	15.019
1989 - 90	5.154	4.065	2.692	5.704	9.081
1990 - 91	5.889	3.590	2.855	6.398	10.575
Expected Return	**4.560**	**2.192**	**2.084**	**5.521**	**9.055**

Table 1.2: Variance and Co-variance Matrix of Gross Return for Crop Activities, Dhaka Division 1983-84 to 1990-91: In million Taka

Enterprises	Cereals (X_1)	Pulses (X_2)	Oilseeds (X_3)	Fibre (X_4)	Vegetables (X_5)
Cereals (X_1)	0.65	1.07	0.55	0.46	1.19
Pulses (X_2)		2.24	1.04	0.32	2.99
Oilseeds (X_3)			0.62	0.30	1.09
Fibre (X_4)				1.42	- 0.72
Vegetables (X_5)					8.89

CORRELATION OF RETURNS ANALYSIS

Pearson Correlation Coefficients / Prob > |R| under Ho: Rho=0 / N = 7

	X_1	X_2	X_3	X_4	X_5
X_1	1.00000 0.0	0.88585 0.0079	0.86676 0.0116	0.47757 0.2785	0.49492 0.2588
X_2	0.88585 0.0079	1.00000 0.0	0.88371 0.0083	0.18354 0.6936	0.67136 0.0987
X_3	0.86676 0.0116	0.88371 0.0083	1.00000 0.0	0.32521 0.4766	0.46630 0.2916
X_4	0.47757 0.2785	0.18354 0.6936	0.32521 0.4766	1.00000 0.0	-0.20277 0.6628
X_5	0.49492 0.2588	0.67136 0.0987	0.46630 0.2916	-0.20277 0.6628	1.00000 0.0

LINEAR PROGRAMMING SOLUTION

Maximization of expected return

$$\text{Max } 4.56 X_1 + 2.192 X_2 + 2.084 X_3 + 5.521 X_4 + 9.055 X_5$$

Land Constraint

$$X_1 + X_2 + X_3 + X_4 + X_5 \leq 8973.23$$

Nitrogen Constraint

$$17 X_1 + 0.7 X_2 + 2.77 X_3 + 1.57 X_4 + 4.25 X_5 \leq 171021$$

Phosphorous Constraint

$$5.85 X_1 + 1.41 X_2 + 1.61 X_3 + 0.4 X_4 + 2.09 X_5 \leq 56635$$

Potassium Constraint

$$2.13 X_1 + 1.21 X_2 + 1.61 X_3 + 0.57 X_4 + 3.44 X_5 \leq 20994$$

Labour Constraint

$$70 X_1 + 27 X_2 + 41 X_3 + 96 X_4 + 105 X_5 \leq 1742354$$

LINGO ENVIRONMENT SETTING

```
max=4.56*x1+2.192*x2+2.084*x3+5.521*x4+9.055*x5;
x1+x2+x3+x4+x5<=8973.23;
17*x1+0.7*x2+2.77*x3+1.57*x4+4.25*x5<=171021;
5.85*x1+1.41*x2+1.61*x3+0.4*x4+2.09*x5<=56635;
2.13*x1+1.21*x2+1.61*x3+0.57*x4+3.44*x5<=20994;
70*x1+27*x2+41*x3+96*x4+105*x5<=1742354;
```

RESULTS

Global optimal solution found.

Objective value:	**69094.27**
Infeasibilities:	0.000000
Total solver iterations:	2

Variable	Value	Reduced Cost
X1	0.000000	2.881920
X2	0.000000	4.117070
X3	0.000000	4.717613
X4	**3440.387**	0.000000
X5	**5532.843**	0.000000

Row	Slack or Surplus	Dual Price
1	69094.27	1.000000
2	0.000000	4.819125
3	142105.0	0.000000
4	43695.20	0.000000
5	0.000000	1.231359
6	831128.3	0.000000

Maximum expected income: 69094.27 Million Taka

Decision variables at maximum income level,
X1=0
X2=0
X3=0
X4= 3444.38
X5=5532.843

FARM PLANNING UNDER UNCERTAINTY

Farm Plans	I	II	III	IV	V	VI	VII	VIII
Expected Total Gross Return	10000	20000	30000	39929.61[2]	49500	51000	60000	69094.27 (LP)[3]
Variance								261539516 (LP)
Quadratic	2479292	9917168	22313630	39529180	60748850	64610220	103589200	261539516
MOTAD	2543760	10175205	22894333	40557786	62329857	65851995	103588883	261539516
Standard Deviation								16172.18 (LP)
Quadratic	1574.57	3149.15	4723.73	6287.22	7794.15	8038.04	10177.87	16172.18
MOTAD	1594.91	3189.85	4784.80	6368.49	7894.92	8114.92	10177.86	16172.18
Cereals - X_1								000 (LP)
Quadratic	745.94	1491.88	2237.82	2978.51	3692.40	3165.72	000	000
MOTAD	914.46	1828.93	2743.40	3651.43	4526.61	3754.72	000	000
	(10.19)[1]	(20.38)	(30.57)	(40.69)	(50.44)	(41.84)		
Pulses - X_2								000 (LP)
Quadratic	000	000	000	000	000	000	000	000
MOTAD	000	000	000	000	000	000	000	000
Oilseeds - X_3								000 (LP)
Quadratic	000	000	000	000	000	000	000	000
MOTAD	000	000	000	000	000	000	000	000
Fibre - X_4								3440.38 (LP)
Quadratic	833.97	1667.95	2501.92	3330.03	4128.18	4533.86	6013.75	3440.38
MOTAD	648.46	1296.92	1945.38	2589.28	3209.88	3784.69	6013.75	3440.38
	(7.22)	(14.45)	(21.67)	(28.85)	(35.77)	(42.17)	(67.01)	(38.34)
								5532.84 (LP)
Vegetables - X_5	220.22	440.44	660.67	879.34	1090.10	1273.64	2959.47	5532.84
Quadratic	248.46	496.93	745.40	992.11	1229.91	1433.81	2959.47	5532.84
MOTAD	(2.76)	(5.53)	(8.30)	(11.05)	(13.70)	(15.97)	(32.98)	(61.65)

Notes : 1. Figures in parentheses are the percentages of the total cropped area (8973.23 thousands acres) of Dhaka division
2. Prevailing return level of Dhaka division
3. Maximum return level of Dhaka division using the available resources (LP = linear programming solutions).

(Area in '000 Acres and Return in Million Taka)

Farm Plans	I	II	III	IV	V	VI	VII	VIII
Expected Total Gross Return	10000	20000	30000	39929.61	49500	51000	60000	69094.27 (LP)
<u>Total Area</u> Quadratic MOTAD	 1800.13 1811.38 (20.18)	 3600.27 3622.78 (40.37)	 5400.41 5434.18 (60.55)	 7187.88 7232.82 (80.60)	 8910.68 8966.40 (99.92)	 8973.22 8973.22 (100)	 8973.23 8973.22 (100)	8973.22 (LP) 8973.22 8973.22 (100)
Net Cropped Area	5253.28 (58.54)	5253.28 (58.54)	5253.28 (58.54)	5253.28 (58.54)	5253.28 (58.54)	5253.28 (58.54)	5253.28 (58.54)	5253.28 (58.54)
<u>Intensity of Cropping (%)</u> Quadratic MOTAD	 34.26 34.48	 68.53 68.96	 102.80 103.44	 136.82 137.68	 169.62 170.68	 170.81 170.81	 170.81 170.81	170.81 (LP) 170.81 170.81
<u>Coefficient of Variation (%)</u> Quadratic MOTAD	 15.74 15.94	 15.74 15.94	 15.74 15.94	 15.74 15.94	 15.74 15.94	 15.76 15.94	 16.96 16.96	23.40 (LP) 23.40 23.40

Note :
1. Intensity of cropping is measured by dividing the total area by the corresponding net cropped area and multiplied it by 100.
2. Coefficient of variation is measured by dividing the standard deviation by the corresponding expected return and multiplied it by 100

MOTAD Model

Minimization of the Total Absolute Deviations

$$MIN\ YP_{84} + YP_{85} + YP_{86} + YP_{88} + YP_{89} + YP_{90} + YP_{91} + YN_{84} + YN_{85} + YN_{86} + YN_{88} + YN_{89} + YN_{90} + YN_{91}$$

Subject to

Return Constraint

2) $4.560\ X_1 + 2.192\ X_2 + 2.084\ X_3 + 5.521\ X_4 + 9.055\ X_5 = 39929.61$

Land Constraint

3) $X_1 + X_2 + X_3 + X_4 + X_5 \leq 8973.23$

Nitrogen Constraint

4) $17\ X_1 + 0.7\ X_2 + 2.77\ X_3 + 1.57\ X_4 + 4.25\ X_5 \leq 171021$

Phosphorus Constraint

5) $5.85\ X_1 + 1.41\ X_2 + 1.61\ X_3 + 0.4\ X_4 + 2.09\ X_5 \leq 56635$

Potassium Constraint

6) $2.13 X_1 + 1.21 X_2 + 1.61 X_3 + 0.57 X_4 + 3.44 X_5 \leq 20994$

Labour Constraint

7) $70 X_1 + 27 X_2 + 41 X_3 + 96 X_4 + 105 X_5 \leq 1742354$

Risk Rows : 1983- 84 to 1990-91

8) $- 100 YP_{84} + 100 YN_{84} - 108 X_1 - 156.7 X_2 - 86.6 X_3 - 162.4 X_4 - 200 X_5 = 0$

9) $- 100 YP_{85} + 100 YN_{85} - 50.7 X_1 - 154.1 X_2 - 93.4 X_3 + 156.1 X_4 - 220.8 X_5 = 0$

10) $- 100 YP_{86} + 100 YN_{86} - 59.3 X_1 - 120.8 X_2 - 70.5 X_3 - 115.8 X_4 - 87.6 X_5 = 0$

11) $- 100 YP_{88} + 100 YN_{88} + 2.5 X_1 - 23.4 X_2 + 68.3 X_3 + 89.2 X_4 - 242 X_5 = 0$

12) $-100 YP_{89} +100 YN_{89} + 23.7 X_1 + 128.4 X_2 + 44.4 X_3 - 72.5 X_4 + 596.4 X_5 = 0$

13) $- 100 YP_{90} + 100 YN_{90} + 59.4 X_1 + 187.3 X_2 + 60.8 X_3 + 18.3 X_4 + 2.6 X_5 = 0$

14) $- 100 YP_{91} + 100 YN_{91} + 132.9 X_1 + 139.8 X_2 + 77.1 X_3 + 87.7 X_4 + 152 X_5 = 0$

Non- Negativity Constraints

15) $X_1 \geq 0$

16) $X_2 \geq 0$

17) $X_3 \geq 0$

18) $X_4 \geq 0$

19) $X_5 \geq 0$

QUADRATIC PROGRAMMING

Estimation of variance and co-variance matrix

Before a complete model can be constructed, the estimation on variance and covariances are needed. There are a number of ways for measuring the variance. In this study, following formula has been employed for measuring the variance of return of both prevailing and efficient farm plans given in equation 1.1.

$$V = \sum_{j=1}^{n} \sum_{k=1}^{n} X_j X_k \sigma_{jk} \qquad (1.1)$$

The variance of the five crops portfolio is stated as follows :

$$Var(p) = X_1X_1\sigma_{11} + 2X_1X_2\sigma_{12} + 2X_1X_3\sigma_{13} + 2X_1X_4\sigma_{14} + 2X_1X_5\sigma_{15} + X_2X_2\sigma_{22} + 2X_2X_3\sigma_{23} + 2X_2X_4\sigma_{24} + 2X_2X_5\sigma_{25} + X_3X_3\sigma_{33} + 2X_3X_4\sigma_{34} + 2X_3X_5\sigma_{35} + X_4X_4\sigma_{44} + 2X_4X_5\sigma_{45} + X_5X_5\sigma_{55}$$

Var (p)= Variance of the five crop portfolio

$$= X_1^2 (0.65) + 2.14 X_1X_2 + 1.1 X_1X_3 + 0.92 X_1X_4 + 2.38 X_1X_5 + X_2^2 (2.24) + 2.08 X_2X_3 + 0.64 X_2X_4 + 5.98 X_2X_5 + X3^2 (0.62) + 0.6 X_3X_4 + 2.18 X_3X_5 + X_1^2 (1.42) – 1.44 X_4X_5 + X_5^2 (8.89)$$

First Order condition for X1

$\delta V/\delta X1 == X_1^2\,(0.65) + 2.14\,X_1X_2 + 1.1\,X_1X_3 + 0.92\,X_1X_4 + 2.38\,X_1X_5 + X_2^2\,(2.24) + 2.08\,X_2X_3 + 0.64\,X_2X_4 + 5.98\,X_2X_5 + X3^2\,(0.62) + 0.6\,X_3X_4 + 2.18\,X_3X_5 + X_1^2\,(1.42) - 1.44\,X_4X_5 + X_5^2\,(8.89)$

$= 1.3\,X_1 + 2.14\,X_2 + 1.1\,X_3 + 0.92\,X_4 + 2.38\,X_5$

LINDO APPLICATION

A Quadratic Programming solution for Dhaka Division

Minimization of Variance

MIN $X_1 + X_2 + X_3 + X_4 + X_5 + A + B + C + D + E + F$

<u>Subject to</u>

First order condition for X_1

2) $1.3\,X_1 + 2.14\,X_2 + 1.1\,X_3 + 0.92\,X_4 + 2.38\,X_5 + A - 4.56\,B + 17\,C + 5.85\,D$

$+ 2.13\,E + 70\,F \geq 0$

First order condition for X_2

3) $2.14 X_1 + 4.48 X_2 + 2.08 X_3 + 0.64 X_4 + 5.98 X_5 + A - 2.192 B + 0.7 C + 1.41 D$

$+ 1.21 E + 27 F \geq 0$

First order condition for X_3

4) $1.1 X_1 + 2.08 X_2 + 1.24 X_3 + 0.6 X_4 + 2.18 X_5 + A - 2.084 B + 2.77 C + 1.61 D$

$+ 1.61E + 41 F \geq 0$

First order condition for X_4

5) $0.92 X_1 + 0.64 X_2 + 0.6 X_3 + 2.84 X_4 - 1.44 X_5 + A - 5.521 B + 1.57 C + 0.4 D$

$+ 0.57 E + 96 F \geq 0$

First order condition for X_5

6) $2.38 X_1 + 5.98 X_2 + 2.18 X_3 - 1.44 X_4 + 17.78 X_5 + A - 9.055 B + 4.25 C + 2.09 D$

$+ 3.44 E + 105 F \geq 0$

Land Constraint : Multiplier is A

7) $X_1 + X_2 + X_3 + X_4 + X_5 \leq 8973.23$

Expected Return Constraint : Multiplier is B

8) $4.56 X_1 + 2.192 X_2 + 2.084 X_3 + 5.521 X_4 + 9.055 X_5 \geq 39929.61$

Nitrogen Constraint : Multiplier is C

9) $17 X_1 + 0.7 X_2 + 2.77 X_3 + 1.57 X_4 + 4.25 X_5 \leq 171021$

Phosphorous Constraint : Multiplier is D

10) $5.85 X_1 + 1.41 X_2 + 1.61 X_3 + 0.4 X_4 + 2.09 X_5 \leq 56635$

Potassium Constraint : Multiplier is E

11) $2.13 X_1 + 1.21 X_2 + 1.61 X_3 + 0.57 X_4 + 3.44 X_5 \leq 20994$

Labour Constraint : Multiplier is F

12) $70 X_1 + 27 X_2 + 41 X_3 + 96 X_4 + 105 X_5 \leq$

END

QCP=6

Prepared by:
Sayed Hossain
Faculty of Management, Multimedia University, 63100 Cyberjaya, Malaysia
Personal Website: www.sayedhossain.com
Email: sayed@sayedhossain.com

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