STOCK PORTFOLIO MODEL
MODEL ONE

Keywords: Decision making under uncertainty, efficient portfolio, variance analysis

INCOME AND RISK (VARIANCE) OF A ASSET

- A portfolio contains a number of assets. Each asset is associated with income and risk or uncertainty. Suppose X1 is a stock in a portfolio. The mean of the historical income of X1 is its mean or expected income and the standard deviation (square root of variance) of the income is risk or uncertainty or volatility of this asset X1. The more the standard deviation, more the risky X1 is.

RISK OF A PORTFOLIO

- Like risk of a particular asset like X1, we can also calculate the risk of a whole portfolio which contains many assets like X1, X2, X3. . . . . Xn.
- So the risk of a portfolio or variance of income (V) of a portfolio can be calculated as follows. Equation No. (1.1)

\[ V = \sum_{j=1}^{n} \sum_{k=1}^{n} X_j X_k \sigma_{jk} \]  

(1.1)
When there are five assets in a portfolio, then risk of the portfolio would be as follows:

\[
\text{Variance} = \sum_{j=1}^{5} X_j \sigma_{jj} + \sum_{j=1}^{5} \sum_{k=1}^{5} X_j X_k \rho_{jk} \sigma_j \sigma_k
\]

Since \( \sigma_{jk} = \rho_{jk} \sigma_j \sigma_k \) for \( j \neq k \),

Where,

- \( \text{V} \) is the variance of income of the stock portfolio.
- \( X_j \) is the amount allocated or decision variable.
- \( \sigma_{jk} \) is the covariance of gross income between the \( j \)th and \( k \)th enterprises.
\( \rho_{jk} \) is the correlation coefficient between jth and kth enterprise.

\( \sigma_{ij} \) is the variance of income of the jth enterprise or covariance of the jth variable with itself.

\( \sigma_j \) is the standard deviation of income of the jth enterprise.

\( \sigma_k \) is the standard deviation of income of the kth enterprise.

**CALCULATION OF THE NUMBER OF VARIANCES AND CO-VARIANCES OF A PORTFOLIO**

\( n = \) number of variances of return of a portfolio

\( (n^2 - n) = \) number of co-variances in a portfolio

Total variances and co-variances in a 5 asset portfolio as follows.

Here \( n = 5 \) (variances as well as number of assets)

\[ = n + (n^2 - n) \]

\[ = 5 + (25 - 5) = 25 \]

Here, \( 5 = \) variances and \( 20 = \) co-variances.
HISTORICAL ANNUAL INCOME

- Assume the following one is a historical income from five stocks in million dollars

Historical income of three stocks such as X1, X2 and X3

<table>
<thead>
<tr>
<th>Year</th>
<th>X1</th>
<th>X2</th>
<th>X3</th>
</tr>
</thead>
<tbody>
<tr>
<td>1991</td>
<td>3.480</td>
<td>0.625</td>
<td>3.897</td>
</tr>
<tr>
<td>1992</td>
<td>4.053</td>
<td>0.651</td>
<td>7.082</td>
</tr>
<tr>
<td>1993</td>
<td>3.967</td>
<td>0.984</td>
<td>4.363</td>
</tr>
<tr>
<td>1994</td>
<td>4.585</td>
<td>1.958</td>
<td>6.413</td>
</tr>
<tr>
<td>1995</td>
<td>4.797</td>
<td>3.476</td>
<td>4.796</td>
</tr>
<tr>
<td>1996</td>
<td>5.154</td>
<td>4.065</td>
<td>5.704</td>
</tr>
<tr>
<td>1997</td>
<td>5.889</td>
<td>3.590</td>
<td>6.398</td>
</tr>
</tbody>
</table>

Mean income | 4.560 | 2.192 | 5.521
### DESCRIPTIVE STATISTICS (SPSS)
(From SPSS)

#### Descriptive Statistics

<table>
<thead>
<tr>
<th></th>
<th>N</th>
<th>Minimum</th>
<th>Maximum</th>
<th>Mean</th>
<th>Std. Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>7</td>
<td>3.48000</td>
<td>5.88900</td>
<td>4.5607143</td>
<td>.81147658</td>
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<tr>
<td>x2</td>
<td>7</td>
<td>.62500</td>
<td>4.06500</td>
<td>2.1927143</td>
<td>1.49751881</td>
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<tr>
<td>x3</td>
<td>7</td>
<td>3.89700</td>
<td>7.08200</td>
<td>5.5218571</td>
<td>1.19296290</td>
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<tr>
<td>Valid N (listwise)</td>
<td>7</td>
<td></td>
<td></td>
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</tbody>
</table>

### VARIANCE, COVARIANCE AND CORRELATION ANALYSIS
(From SPSS)

#### Correlations

<table>
<thead>
<tr>
<th></th>
<th>x1</th>
<th>x2</th>
<th>x3</th>
</tr>
</thead>
<tbody>
<tr>
<td>x1</td>
<td>1</td>
<td>.886**</td>
<td>.478</td>
</tr>
<tr>
<td>x2</td>
<td>.008</td>
<td>.278</td>
<td></td>
</tr>
<tr>
<td>x3</td>
<td>3.951</td>
<td>6.459</td>
<td>2.774</td>
</tr>
<tr>
<td>Covariance</td>
<td>.658</td>
<td>1.076</td>
<td>.462</td>
</tr>
<tr>
<td>N</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

*Significant at .05 level (two-tailed)*

**Significant at .01 level (two-tailed)
<table>
<thead>
<tr>
<th></th>
<th>Pearson Correlation</th>
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<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>x2</td>
<td>.886**</td>
<td>1</td>
<td>.184</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sig. (2-tailed)</td>
<td>.008</td>
<td>.694</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Sum of Squares and</td>
<td>6.459</td>
<td>13.455</td>
<td>1.967</td>
</tr>
<tr>
<td></td>
<td>Cross-products</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Covariance</td>
<td>1.076</td>
<td>2.243</td>
<td>.328</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>7</td>
<td>7</td>
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<tr>
<td></td>
<td>Cross-products</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Covariance</td>
<td>.462</td>
<td>.328</td>
<td>1.423</td>
</tr>
<tr>
<td></td>
<td>N</td>
<td>7</td>
<td>7</td>
<td>7</td>
</tr>
</tbody>
</table>

**. Correlation is significant at the 0.01 level (2-tailed).
VARIANCE (RISK) OF THREE ASSETS PORTFOLIO

Variance of a portfolio  \( = \sum \sum X_j X_k \sigma_{jk} \)

\( j=1 \) \( k=1 \)

Varince of three stock portfolio.

\[
\begin{align*}
\text{Variance} & = X_1X_1\sigma_{11} + X_1X_2\sigma_{12} + X_1X_3\sigma_{13} \\
& + X_2X_1\sigma_{21} + X_2X_2\sigma_{22} + X_2X_3\sigma_{23} \\
& + X_3X_1\sigma_{31} + X_3X_2\sigma_{32} + X_3X_3\sigma_{33}
\end{align*}
\]

Here \( n=3 \) (variances as well as number of assets)

\( = n + (n^2-n) \)

\( = 3 + (9-3) = 9 \)

Here, 3 variances and 6 co-variances of income.

Variance of 3 assets portfolio  \( = X_1X_1\sigma_{11} + 2X_1X_2\sigma_{12} + 2X_1X_3\sigma_{13} + X_2X_2\sigma_{22} + 2X_2X_3\sigma_{23} + X_3X_3\sigma_{33} \)
LINEAR PROGRAMMING SOLUTION

Maximization of income from three assets.

Max $4.56 \times X_1 + 2.192 \times X_2 + 5.521 \times X_3$

Subject to

Total allocation is 1.

$X_1 + X_2 + X_3 \leq 1$

LINGO ENVIRONMENT SETTING

$max=4.56*x1+2.192*x2+5.521*x3;$

$x1+x2+x3<=1;$
RESULTS OF MAXIMIZATION OF INCOME

Global optimal solution found.
Objective value: 5.521000
Total solver iterations: 0

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Reduced Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>X1</td>
<td>0.000000</td>
<td>0.9610000</td>
</tr>
<tr>
<td>X2</td>
<td>0.000000</td>
<td>3.3290000</td>
</tr>
<tr>
<td>X3</td>
<td>1.000000</td>
<td>0.0000000</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Row</th>
<th>Slack or Surplus</th>
<th>Dual Price</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>5.521000</td>
<td>1.000000</td>
</tr>
<tr>
<td>2</td>
<td>0.000000</td>
<td>5.521000</td>
</tr>
</tbody>
</table>

COMMENTS

Maximum income can be achieved 5.521 million dollar from this three assets portfolio if all allocation is devoted to X3.

Decision variables at maximum income level which is 5.521 million dollar

X1=0
X2=0
X3=1
MINIMIZATION OF RISK OR VARIANCE OR STANDARD DEVIATION OF PORTFOLIO

- We need MOTAD (Minimization of total absolute deviation) technique to minimize the risk of a portfolio. Indeed, MOTAD model is an alternative of quadratic programming to estimate efficient frontier.

MOTAD MODEL SETTING WHEN THREE ASSETS (STOCK) IN A PORTFOLIO

Minimization of the total absolute deviations

\[
\text{MIN } P_{91} + P_{92} + P_{93} + P_{94} + P_{95} + P_{96} + P_{97} + N_{91} + N_{92} + N_{93} + N_{94} + N_{95} + N_{96} + N_{97}
\]

Subject to

Expected income from 3 assets business

\[
4.560 X_1 + 2.192 X_2 + 5.521 X_3 = 5.521 \text{ million (Maximum income)}
\]

Allocation of weight

\[
X_1 + X_2 + X_3 \leq 1 \text{ (Total weight)}
\]

Risk Rows: 1991 to 1997

- \( P_{91} + N_{91} - 1.08x1 - 1.567x2 - 1.62x3 = 0; \)
- \( P_{92} + N_{92} - 0.507x1 - 1.541x2 + 1.56x3 = 0; \)
- \( P_{93} + N_{93} - 0.593x1 - 1.208x2 - 1.15x3 = 0; \)
- \( P_{94} + N_{94} + 0.025x1 - 0.234x2 + 0.892x3 = 0; \)
-P95+N95+0.237x1+1.284x2-0.725x3=0;
-P96+N96+0.594x1+1.873x2+0.183x3=0;
-P97+N97+1.329x1+1.398x2+0.87x3=0;

Non- negativity constraints

\[ X_1 \geq 0 \]
\[ X_2 \geq 0 \]
\[ X_3 \geq 0 \]

**BUSINESS PLAN UNDER RISK AND UNCERTAINTY**

<table>
<thead>
<tr>
<th>Business Plan</th>
<th>I</th>
<th>II</th>
<th>III (Maximum income)</th>
<th>IV (beyond maximum)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Expected income</td>
<td>2 million</td>
<td>4 million</td>
<td>5.521</td>
<td>5.523</td>
</tr>
<tr>
<td>Total risk (variance)</td>
<td>0.108</td>
<td>0.434</td>
<td>1.423</td>
<td></td>
</tr>
<tr>
<td>Total risk (std.dev)</td>
<td>0.328</td>
<td>0.658</td>
<td>1.19</td>
<td>No feasible solution</td>
</tr>
<tr>
<td>X1</td>
<td>0.314</td>
<td>0.629</td>
<td>00</td>
<td></td>
</tr>
<tr>
<td>X2</td>
<td>00</td>
<td>00</td>
<td>00</td>
<td></td>
</tr>
<tr>
<td>X3</td>
<td>0.1022</td>
<td>0.2045</td>
<td>1.00</td>
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</tr>
<tr>
<td>CV</td>
<td></td>
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</tr>
</tbody>
</table>
MINIMUM VARIANCE PORTFOLIO

Expected income

Minimum Risk Efficient frontier

Standard Deviation

2 million

4 million

5.521 million

0.328

0.658

1.19

0.328

0.658

1.19

Standard Deviation
GUIDELINE

- The target of MOTAD model is to reduce the variance of the portfolio.

- MOTAD model will be looking for those stocks which are negatively correlated or less correlated to reduce the variance of the portfolio.

END

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