STOCK POTFOLIO MODEL
MODEL ONE

Keywords: Decision making under uncertainty, efficient portfolio, variance analysis

INCOME AND RISK (VARIANCE) OF A ASSET

- A portfolio contains a number of assets. Each asset is associated with income and risk or uncertainty. Suppose X1 is a stock in a portfolio. The mean of the historical income of X1 is its mean or expected income and the standard deviation (square root of variance) of the income is risk or uncertainty or volatility of this asset X1. The more the standard deviation, more the risky X1 is.

RISK OF A PORTFOLIO

- Like risk of a particular asset like X1, we can also calculate the risk of a whole portfolio which contains many assets like X1, X2, X3. …... Xn.
- So the risk of a portfolio or variance of income (V) of a portfolio can be calculated as follows. Equation No. (1.1)

$$V = \sum_{j=1}^{n} \sum_{k=1}^{n} X_j \; X_k \; \sigma_{jk} \qquad (1.1)$$

When there are five assets in a portfolio, then risk of the portfolio would be as follows:

$$\text{Variance} = \begin{array}{ccccccccc} X_1X_1\sigma_{11} & + & X_1X_2\sigma_{12} & + & X_1X_3\sigma_{13} & + & X_1X_4\sigma_{14} & + & X_1X_5\sigma_{15} \\ X_2X_1\sigma_{21} & + & X_2X_2\sigma_{22} & + & X_2X_3\sigma_{23} & + & X_2X_4\sigma_{24} & + & X_2X_5\sigma_{25} \\ X_3X_1\sigma_{31} & + & X_3X_2\sigma_{32} & + & X_3X_3\sigma_{33} & + & X_3X_4\sigma_{34} & + & X_3X_5\sigma_{35} \\ X_4X_1\sigma_{41} & + & X_4X_2\sigma_{42} & + & X_4X_3\sigma_{43} & + & X_4X_4\sigma_{44} & + & X_4X_5\sigma_{45} \\ X_5X_1\sigma_{51} & + & X_5X_2\sigma_{52} & + & X_5X_3\sigma_{53} & + & X_5X_4\sigma_{54} & + & X_5X_5\sigma_{55} \end{array}$$

Variance $= X_1X_1\sigma_{11} + 2\,X_1X_2\sigma_{12} + 2X_1X_3\sigma_{13} + 2X_1X_4\sigma_{14} + 2\,X_1X_5\sigma_{15} + X_2X_2\sigma_{22} + 2X_2X_3\sigma_{23} + 2X_2X_4\sigma_{24} + 2X_2X_5\sigma_{25} + X_3X_3\sigma_{33} + 2X_3X_4\sigma_{34} + 2\,X_3X_5\sigma_{35} + X_4X_4\sigma_{44} + 2X_4X_5\sigma_{45} + X_5X_5\sigma_{55}$

$$= \sum_{j=1}^{5} X^2_j\,\sigma_{jj} + \sum_{j=1}^{5}\sum_{k=1}^{5} X_j\ X_k\ \sigma_{jk}$$

$$= \sum_{j=1}^{5} X^2_j\,\sigma_{jj} + \sum_{\substack{j=1 \\ \text{for } j \neq k}}^{5}\sum_{k=1}^{5} X_j\,X_k\,\rho_{jk}\,\sigma_j\,\sigma_k \qquad \text{since} \quad \sigma_{jk} = \rho_{jk}\,\sigma_j\,\sigma_k$$

Where,

V is the variance of income of the stock portfolio

X_j is the amount allocated or decision variable.

σ_{jk} is the covariance of gross income between the jth and kth enterprises.

ρ_{jk} is the correlation coefficient between jth and kth enterprise.

σ_{jj} is the variance of income of the jth enterprise or covariance of the jth variable with itself.

σ_j is the standard deviation of income of the jth enterprise.

σ_k is the standard deviation of income of the kth enterprise.

CALCULATION OF THE NUMBER OF VARIANCES AND CO-VARIANCES OF A PORTFOLIO

n= number of variances of return of a portfolio

(n^2-n) = number of co-variances in a portfolio

Total variances and co-variances in a 5 asset portfolio as follows.

Here n=5 (variances as well as number of assets)

$= n + (n^2-n)$

$= 5 + (25-5) = 25$

Here, 5 = variances and 20 co-variances.

HISTORICAL ANNUAL INCOME

- Assume the following one is a historical income from five stocks in million dollar

Historical income of three stocks such as X1, X2 and X3

Year	X1	X2	X3
1991	3.480	0.625	3.897
1992	4.053	0.651	7.082
1993	3.967	0.984	4.363
1994	4.585	1.958	6.413
1995	4.797	3.476	4.796
1996	5.154	4.065	5.704
1997	5.889	3.590	6.398
Mean income	**4.560**	**2.192**	**5.521**

DESCRIPTIVE STATISTICS (SPSS)
(From SPSS)

Descriptive Statistics

	N	Minimum	Maximum	**Mean**	**Std. Deviation**
x1	7	3.48000	5.88900	4.5607143	.81147658
x2	7	.62500	4.06500	2.1927143	1.49751881
x3	7	3.89700	7.08200	5.5218571	1.19296290
Valid N (listwise)	7				

VARIANCE, COVARIANCE AND CORRELATION ANALYSIS
(From SPSS)

Correlations

		x1	**x2**	**x3**
x1	Pearson Correlation	**1**	**.886********	**.478**
	Sig. (2-tailed)		.008	.278
	Sum of Squares and Cross-products	3.951	6.459	2.774
	Covariance	**.658**	**1.076**	**.462**
	N	7	7	7

x2	Pearson Correlation	.886**	1	.184
	Sig. (2-tailed)	.008		.694
	Sum of Squares and Cross-products	6.459	13.455	1.967
	Covariance	1.076	2.243	.328
	N	7	7	7
x3	Pearson Correlation	.478	.184	1
	Sig. (2-tailed)	.278	.694	
	Sum of Squares and Cross-products	2.774	1.967	8.539
	Covariance	.462	.328	1.423
	N	7	7	7

**. Correlation is significant at the 0.01 level (2-ailed).

VARIANCE (RISK) OF THREE ASSETS PORTFOLIO

Variance of a portfolio $= \sum_{j=1} \sum_{k=1} X_j \; X_k \; \sigma_{jk}$

Varince of three stock portfolio.

$$\text{Variance} = \begin{array}{ccccc} X_1X_1\sigma_{11} & + & X_1X_2\sigma_{12} & + & X_1X_3\sigma_{13} \\ X_2X_1\sigma_{21} & + & X_2X_2\sigma_{22} & + & X_2X_3\sigma_{23} \\ X_3X_1\sigma_{31} & + & X_3X_2\sigma_{32} & + & X_3X_3\sigma_{33} \end{array}$$

Here n=3 (variances as well as number of assets)

$= n + (n^2-n)$

$= 3 + (9-3) = 9$

Here, 3 variances and 6 co-variances of income.

Variance of 3 assets portfolio $= X_1X_1\sigma_{11} + 2\,X_1X_2\sigma_{12} + 2X_1X_3\sigma_{13} + X_2X_2\sigma_{22} + 2X_2X_3\sigma_{23} + X_3X_3\sigma_{33}$

LINEAR PROGRAMMING SOLUTION

Maximization of income from three assets.

Max $4.56\, X_1 + 2.192\, X_2 + 5.521\, X_3$

Subject to

Total allocation is 1.

$X_1 + X_2 + X_3 \leq 1$

LINGO ENVIRONMENT SETTING

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max=4.56*x1+2.192*x2+5.521*x3;

x1+x2+x3<=1;
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RESULTS OF MAXIMIZATION OF INCOME

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Global optimal solution found.
Objective value:                              5.521000
Total solver iterations:                             0
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Variable	Value	Reduced Cost
X1	0.000000	0.9610000
X2	0.000000	3.329000
X3	**1.000000**	0.000000

Row	Slack or Surplus	Dual Price
1	5.521000	1.000000
2	0.000000	5.521000

COMMENTS

Maximum income can be achieved 5.521 million dollar from this three assets portfolio if all allocation is devoted to X3.

Decision variables at maximum income level which is 5.521 million dollar

X1=0
X2=0
X3=1

MINIMIZATION OF RISK OR VARIANCE OR STANDARD DEVIATION OF PORTFOLIO

- We need MOTAD (Minimization of total absolute deviation) technique to minimize the risk of a portfolio. Indeed, MOTAD model is an alternative of quadratic programming to estimate efficient frontier.

MOTAD MODEL SETTING WHEN THREE ASSETS (STOCK) IN A PORTFOLIO

Minimization of the total absolute deviations

MIN P91 + P92 + P93 + P94 + P95 + P96 + P97 + N91 + N92 + N93 + N94 + N95 + N96 + N97

Subject to

Expected income from 3 assets business

$4.560\ X_1 + 2.192\ X_2 + 5.521\ X_3 = 5.521$ million (Maximum income)

Allocation of weight

$X_1 + X_2 + X_3 \leq 1$ (Total weight)

Risk Rows : 1991 to 1997

-P91+N91-1.08x1-1.567x2-1.62x3=0;

-P92+N92-0.507x1-1.541x2+1.56x3=0;

-P93+N93-0.593x1-1.208x2-1.15x3=0;

-P94+N94+0.025x1-0.234x2+0.892x3=0;

-P95+N95+0.237x1+1.284x2-0.725x3=0;

-P96+N96+0.594x1+1.873x2+0.183x3=0;

-P97+N97+1.329x1+1.398x2+0.87x3=0;

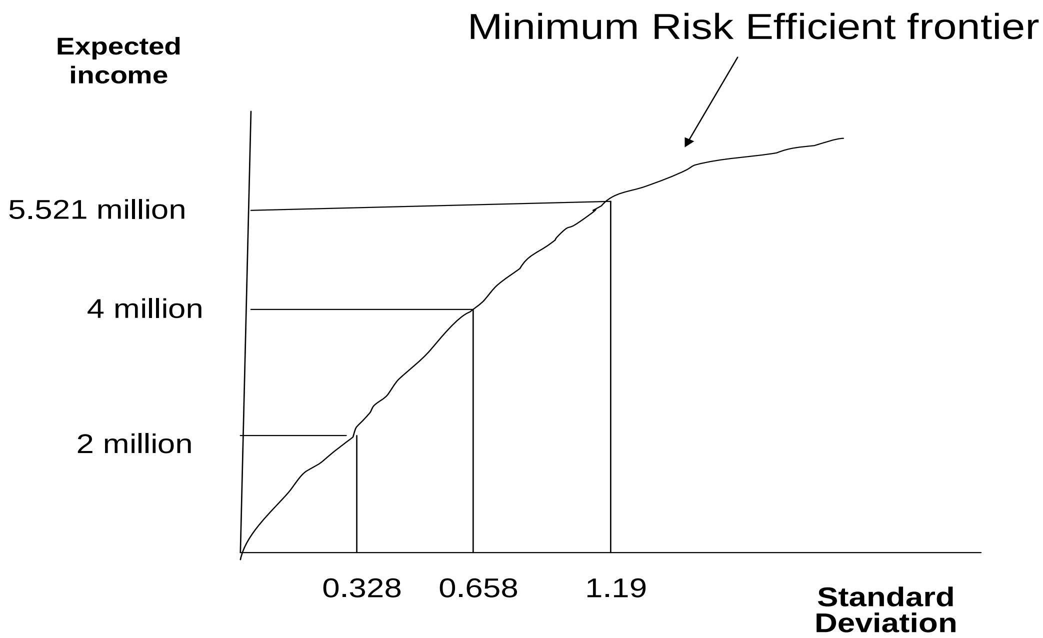
Non- negativity constraints

X_1>=0
X_2>=0
X_3>=0

BUSINESS PLAN UNDER RISK AND UNCERTAINTY

Business Plan	**I**	**II**	**III (Maximum income)**	**IV (beyond maximum)**
Expected income	2 million	4 million	5.521	5.523
Total risk (variance)	0.108	0.434	1.423	
Total risk (std.dev)	0.328	0.658	1.19	No feasible solution
X1	0.314	0.629	00	
X2	00	00	00	
X3	0.1022	0.2045	1.00	
CV				

MINIMUM VARIANCE PORTFOLIO



GUIDELINE

- The target of MOTAD model is to reduce the variance of the portfolio.

- MOTAD model will be looking for those stocks which are negatively correlated or less correlated to reduce the variance of the portfolio.

END

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